6 Online Appendix B: Extensions of the Baseline Model

In this appendix, we show that adding decreasing or increasing returns to scale to our model does not change the underlying source of firm inefficiency, that decreasing returns to scale make inefficiency in the firm equilibrium more likely, that there is no inefficiency when the parameters of inventive opportunities tomorrow do not depend on which inventions are discovered today, that a single element of state dependence in conjunction with multiple research lines generates inefficiency, and that permitting both short-lived and infinite-lived research firms exacerbates the racing distortion.

6.1 Planner Problem with Nonlinear Hazard Rates

First, consider alternative assumptions about returns to scale. Let the hazard rate on invention k for firm i be $\lambda_k h(x_k)$, where h is twice-differentiable, $h' > 0$, $h(0) = 0$ and, without loss of generality, $h(\frac{1}{N}) = \frac{1}{N}$. Under decreasing returns to scale, $h'' < 0$, and under increasing returns, $h'' > 0$. Note that, in the results presented in the body of this paper, constant returns to scale under the above assumptions simply means that $h(x) = x$. To simplify notation, throughout this section we assume that there is no inefficiency in future states.

In section 5.2.1 in Appendix A, we showed that independence of hazard rates across firms means the planner optimizes with symmetric effort across firms. Without loss of generality, we assume $M = 1$, so the planner solves

$$\max_{\sum\limits_{s' \in S(s)} x_{s'} \le \frac{1}{N},\ x_{s'} \ge 0, \forall s' \in S(s)} \frac{\sum_{s'} \lambda_{s'} P_{s'} N h(x_{s'})}{r + \sum_{s'} \lambda_{s'} N h(x_{s'})}$$

The KKT necessary condition imply that exist $\mu_{s'} \ge 0$ such that $\mu_{s'} x_{s'} = 0$ and γ such that

$$\frac{\partial f(x)}{\partial x_{s'}} = \gamma - \mu_{s'}.$$

A corner solution, where all effort goes to $k \in S(s)$, that is $x_k = \frac{1}{N}$ and $x_\ell = 0$ for $\ell \neq k$

is characterized by

$$\frac{\lambda_k P_k h'(x_k)(r_N + \sum_{s'} \lambda_{s'} h(x_{s'})) - \lambda_k h'(x_k)(\sum_{s'} \lambda_{s'} P_{s'} h(x_{s'}))}{(r_N + \sum_{s'} \lambda_{s'} h(x_{s'}))^2} \geq$$
$$\frac{\lambda_\ell P_\ell h'(x_\ell)(r_N + \sum_{s'} \lambda_{s'} h(x_{s'})) - \lambda_\ell h'(x_\ell)(\sum_{s'} \lambda_{s'} P_{s'} h(x_{s'}))}{(r_N + \sum_{s'} \lambda_{s'} h(x_{s'}))^2}$$

where $r_N = \frac{r}{N}$. Using that $h(0) = 0$, this simplifies to

$$\begin{aligned} \lambda_k P_k h'(x_k)(r_N + \lambda_k h(x_k)) - \lambda_k h'(x_k)\lambda_k P_k h(x_k) &\geq \\ \lambda_\ell P_\ell h'(0)(r_N + \lambda_k h(x_k)) - \lambda_\ell h'(0)\lambda_k P_k h(x_k) \end{aligned}$$

Let $C = \frac{h'(\frac{1}{N})}{h'(0)}$. Note that under decreasing returns to scale, $C \in (0, 1)$. Thus, we can write

$$\lambda_k P_k C(r_N + \lambda_k h(x_k)) - \lambda_k C \lambda_k P_k h(x_k) \geq \lambda_\ell P_\ell (r_N + \lambda_k h(x_k)) - \lambda_\ell \lambda_k P_k h(x_k)$$

Using that $h(x_k) = \frac{1}{N}$ and rearranging terms, and defining $\Delta_C(k, \ell) = \dfrac{\lambda_\ell - C\lambda_k}{r + \lambda_k}$, we get

$$\lambda_k P_k C \geq \lambda_\ell P_\ell - \Delta_C(k, \ell)\lambda_k P_k.$$

Notice that this condition is equivalent to the planner's condition in Proposition 2. Similar derivation for an arbitrary number of scientists M, defining $C(M) = \frac{h'(\frac{M}{N})}{h'(0)}$, gives the same result.

The only caveat is that KKT are only necessary and not sufficient conditions. However, we show that when $h(x) = x$ the only solution is the corner solution x_k and in that case the condition above holds ($C = 1$). Thus, if inequality holds strictly for $C = 1$, it still holds for C close to 1, in which case we have full effort toward a single invention even with nonconstant returns to scale. Thus, even with small levels of decreasing or increasing returns to scale, the planner corner solution is retained.

6.2 Firm Problem with Nonlinear Hazard Rates

Under the assumption that parameters are such that the planner works on a single invention under decreasing returns to scale, we now show that the firms deviate for almost exactly the same reason as under constant returns. Indeed, decreasing returns to scale make it more likely that firms will deviate because minor deviations to new research lines will generate a higher relative hazard rate under decreasing returns than under constant returns, hence exacerbate the racing distortion.

Suppose that all rivals are exerting efforts towards invention k. Recall the firm problem, if all other firms exert full effort towards invention k, is

$$\max_{\sum_{s' \in S(s)} x_{s'} \leq \frac{1}{N}, x_{s'} \geq 0, \forall s' \in S} \frac{\sum_{s'} \lambda_{s'} P_{fs'} h(x_{s'}) + A_k}{\tilde{r} + \sum_{s'} \lambda_{s'} h(x_{s'})}$$

where $A_k = (N-1)\lambda_k h(\frac{1}{N}) V_{fk}$, and $\tilde{r} = r + (N-1)\lambda_k h(\frac{1}{N})$

As in Section 6.1, the first order necessary condition for positive effort on invention k and no effort on any other invention is

$$\frac{\lambda_k P_{fk} h'(x_k)(\tilde{r} + \lambda_k h(x_k)) - \lambda_k h'(x_k)(\lambda_k P_{fk} h(x_k) + A_k)}{(\tilde{r} + \lambda_k h(x_k))^2} \geq$$
$$\frac{\lambda_\ell P_{f\ell} h'(x_\ell)(\tilde{r} + \lambda_k h(x_k)) - \lambda_\ell h'(x_\ell)(\lambda_k P_{fk} h(x_k) + A_k)}{(\tilde{r} + \lambda_k h(x_k))^2}$$

This simplifies to

$$\lambda_k P_{fk} h'(x_k)(\tilde{r} + \lambda_k h(x_k)) - \lambda_k h'(x_k)(\lambda_k P_{fk} h(x_k) + A_k) \geq$$
$$\lambda_\ell P_{f\ell} h'(x_\ell)(\tilde{r} + \lambda_k h(x_k)) - \lambda_\ell h'(x_\ell)(\lambda_k P_{fk} h(x_k) + A_k)$$

Retaining the assumptions that $h(\frac{1}{N}) = \frac{1}{N}$ and $C = \frac{h'(\frac{1}{N})}{h'(0)}$, after simple algebra we get

$$\lambda_k P_{fk} C \geq \lambda_\ell P_{f\ell} + \frac{1}{N}\Delta_C(k,\ell)\lambda_k P_{fk} - \frac{1}{N}\Delta_C(k,\ell)(N-1)\lambda_k V_{fk}$$

Adding and subtracting terms, we get

$$\lambda_k P_k C \geq \lambda_\ell P_\ell - \Delta_C(k,\ell)\lambda_k P_k + D^*$$

where

$$D^* = \lambda_\ell(P_{f\ell} - P_\ell) - \lambda_k C(P_{fk} - P_k) + \frac{1}{N}\Delta_C(k,\ell)\lambda_k(P_k - (P_{fk} + (N-1)V_{fk})) + \frac{N-1}{N}\Delta_C(k,\ell)\lambda_k P_k$$

This distortion are analogous to the distortions in Proposition 2, with

$$\begin{aligned} D_1^C(k,\ell) &= \lambda_\ell(P_{f\ell} - P_\ell) - \lambda_k C(P_{fk} - P_k) \\ D_2^C(k,\ell) &= \frac{N-1}{N}\Delta_C(k,\ell)\lambda_k P_k \\ D_3^C(k,\ell) &= \frac{1}{N}\Delta_C(k,\ell)\lambda_k(P_k - (P_{fk} + (N-1)V_{fk})) \end{aligned}$$

Thus, adding small amounts of increasing or decreasing returns to scale does not change our main qualitative results.

6.3 Graphs Without State Dependence Have an Efficient Equilibrium

The decomposition in Proposition 2 allows a simple categorization of the nature of inefficiency generated by a particular policy in a particular *type* of invention graph. Inefficiency in the baseline case does not result from the simple existence of multiple projects. Rather, in order to generate inefficiency in the baseline case, a necessary though not sufficient condition is that one firm's actions today must affect the existence of future research targets, or their value, or the difficulty of inventing them. This can be seen with the following simple cases.

First, let there be a set of research targets which are *technologically independent.*

Definition 8. *An invention graph involves* technologically independent inventions *if, in every state, the set of research targets $S(s)$ includes every invention in $S(s_0)$ which has yet to be invented, and the payoff π and simplicity λ of each undiscovered invention never change.*

With technological independence, no matter what is invented today, the options available to inventors tomorrow, and the simplicity and payoff of those inventions, does not change; there is nothing resembling a set of research lines, where invention today affects the nature of inventive opportunity tomorrow. As a result, Proposition 7 shows that on the technologically independent graph, the baseline firm equilibrium is efficient.

Proposition 7. *In an invention graph with technologically independent inventions, the planner optimally works on inventions in decreasing order of their immediate flow social payoff $\lambda_{s'}\pi_{s'}$. Further, there exists an* efficient *firm equilibrium under the baseline policy.*

Proof. We prove by induction. Let there be two remaining inventions. If invention i is discovered first, the expected discounted continuation value for the planner is $V_p(i) =$

$\frac{\lambda_{-i}}{r+\lambda_{-i}}\pi_{-i}$. By Proposition 1, the planner works on invention i that node maximizes the index

$$\frac{\lambda_i}{r+M\lambda_i}[\pi_i + V_i]$$

Define $p_i = \frac{\lambda_i}{r+M\lambda_i}$. The planner discovers 1 first and 2 second if and only if

$$\left(\frac{p_1}{1-p_1}\right)\pi_1 \geq \left(\frac{p_2}{1-p_2}\right)\pi_2.$$

Using the definition of p_i, that inequality simplifies to

$$\lambda_1\pi_1 \geq \lambda_2\pi_2.$$

Now we prove the inductive step. Without loss of generality let $\lambda_1\pi_1 \geq \lambda_2\pi_2 \geq ... \geq \lambda_K\pi_K$. Define $p_i = \frac{\lambda_i}{r+\lambda_i}$ and notice that $\frac{p_i}{1-p_i} = \frac{\lambda_i}{r}$.

We know the result holds for $K = 2$. Assume the result is true for any set of $K-1$ inventions (Induction Hypothesis). Let's prove the result for K inventions. We need to show that starting from 1 is better than starting from any other invention k. By the characterization result, we start from 1 instead of k iff:

$$p_1(\pi_1 + V_p(1)) \geq p_k(\pi_k + V_p(k)), \quad \text{for all } k.$$

Since after one invention there are $K-1$ left, using the induction hypothesis we know that the planner discovers in decreasing order of $\lambda\pi$. Hence,

$$V_p(1) = \sum_{m=2}^{K}\left(\prod_{j=2}^{m} p_j\right)\pi_m \quad \text{and } V_p(k) = \sum_{m=1}^{k-1}\left(\prod_{j=1}^{m} p_j\right)\pi_m + \sum_{m=k+1}^{K}\left(\prod_{j=1,\ j\neq k}^{m} p_j\right)\pi_m.$$

Thus, the condition is equivalent to

$$\sum_{m=1}^{K}\left(\prod_{j=1}^{m} p_j\right)\pi_m \geq p_k\pi_k + p_k\sum_{m=1}^{k-1}\left(\prod_{j=1}^{m} p_j\right)\pi_m + \sum_{m=k+1}^{K}\left(\prod_{j=1}^{m} p_j\right)\pi_m, \quad \text{for all } k.$$

Notice that the terms from $k+1$ to K cancel out. This is because the expected time at which we reach invention $k+1$ is the same if we start from 1 or from k. Thus, we get

$$\sum_{m=1}^{k}\left(\prod_{j=1}^{m} p_j\right)\pi_m \geq p_k\pi_k + p_k\sum_{m=1}^{k-1}\left(\prod_{j=1}^{m} p_j\right)\pi_m.$$

which is equivalent to

$$\sum_{m=1}^{k-1}\left(\prod_{j=1}^{m}p_j\right)\pi_m(1-p_k)\geq p_k\pi_k\left(1-\left(\prod_{j=1}^{k-1}p_j\right)\right).$$

Thus, the planner start from invention 1 if and only if

$$\sum_{m=1}^{k-1}\lambda_m\pi_m\frac{\left(\prod_{j=1}^{m-1}p_j\right)(1-p_m)}{\left(1-\left(\prod_{j=1}^{k-1}p_j\right)\right)}\geq\lambda_k\pi_k,\quad\text{for all } k.$$

This always holds when the inventions are ordered by $\lambda\pi$, since the left hand side of the inequality is a convex combination of $\{\lambda_m\pi_m\}_{m=1}^{k-1}$, since the coefficients

$$a_m=\frac{\left(\prod_{j=1}^{m-1}p_j\right)(1-p_m)}{\left(1-\left(\prod_{j=1}^{k-1}p_j\right)\right)}$$

satisfy that $a_m\geq 0$ and $\sum_{m=1}^{k-1}a_m=1$. The firm equilibrium then follows immediately: since the future is by induction efficient, by Proposition 2 the firms never deviate when the planner is working on the project with highest flow immediate payoff. □

6.4 State Dependent Invention Graphs Generate Inefficiency

Adding an element of state dependence, where invention today affects what can be worked on tomorrow, to the mere existence of multiple projects is enough to induce inefficiency under the baseline policy. Consider a case where all inventions are available in the initial state, but there is no continuation value: once anything has been invented, the immediate social payoff of every other potential invention falls to zero.

Definition 9. *An invention graph involves* perfect substitutes *if all inventions are available in s_0 and any discovery reduces the immediate social payoff of all other inventions to $\pi=0$.*[22]

[22] Our model takes the immediate social payoff of an invention as the reduced form value from an unmodeled demand system. As such, we are in a sense abusing the term "perfect substitutes," but the manner in which the term is used here—two inventions are perfect substitutes if the marginal value of each is zero once the other has been invented—should nonetheless be clear.

With the social continuation value equal to zero, and inventing firms paid exactly the immediate social payoff of their invention, the baseline policy on the perfect substitutes invention graph generates distortions $D_1(s',\ell) = D_3(s',\ell) = 0$, leaving only the racing distortion D_3. Therefore, under perfect substitutes, firms only deviate toward projects which are easier than the planner optimum.

Proposition 8. *Under the baseline policy $\mathcal{P}_{BC}$ on the perfect substitutes invention graph, s' is planner optimal if $\forall \ell \in S(s)$*

$$\lambda_{s'}\pi_{s'} \geq \lambda_\ell \pi_\ell - \lambda_{s'}\pi_{s'}\Delta(s',\ell),$$

and s' is a firm equilibrium under the baseline policy if and only if

$$\lambda_{s'}\pi_{s'} \geq \lambda_\ell \pi_\ell - \lambda_{s'}\pi_{s'}\Delta(s',\ell) + \underbrace{\left(\frac{N-1}{N}\right)\lambda_{s'}\pi_{s'}\Delta(s',\ell)}_{D_2(s',\ell)}.$$

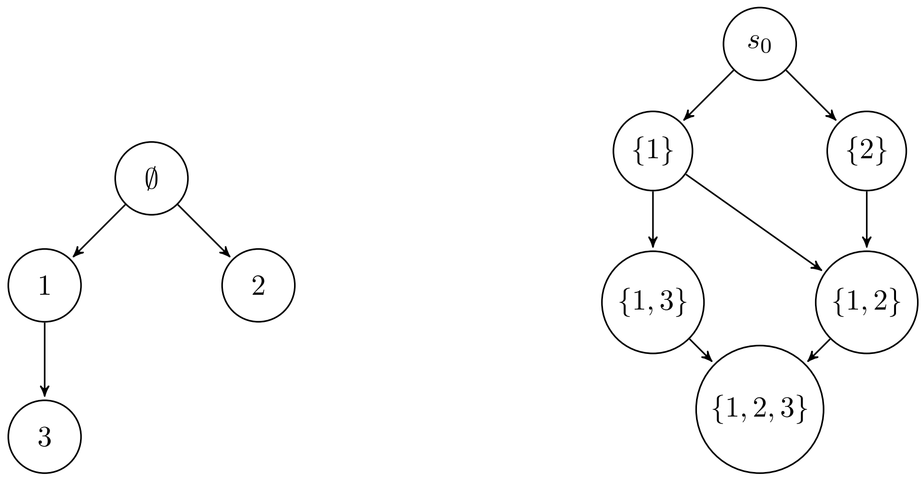
The proof of Proposition 8 is straightforward algebra, hence is omitted.

The technologically independent inventions example shows that equilibrium direction choice is efficient, when all inventions are available from the beginning and there is not state dependency. The perfect substitutes example shows that simple forms of state contingency can generate inefficiency in the equilibrium direction. This case is a particular form of state dependency in parameter values, changing the immediate payoff π. We now show that another type of state contingency, availability of inventions only after other inventions, can also generate directional inefficiencies under the baseline policy.

Consider three inventions. Inventions 1 and 2 are available from the beginning. However, invention 3 becomes available only after 1 is invented. Figure 4a shows the inventions and Figure 4b the states representation.

Proposition 9. *Consider the invention graph in Figure 4. Then:*

1. *If $\lambda_3\pi_3 \leq \max\{\lambda_1\pi_1, \lambda_2\pi_2\}$, then the planner always works on the available invention with largest flow payoff $\lambda\pi$. By Proposition 2 this can be implemented as a firm equilibrium.*



(a) Possible Inventions

(b) Graph of states.

Figure 4: Invention 3 is only available once 1 is invented.

2. *If* $\lambda_3\pi_3 > \max\{\lambda_1\pi_1, \lambda_2\pi_2\}$, *then the planner* opens a path *(works on invention 1 first) iff*
$$\lambda_1\pi_1 \geq \lambda_2\pi_2 + \left(\frac{\lambda_1}{r+\lambda_3}\right)[\lambda_2\pi_2 - \lambda_3\pi_3].$$
Applying the firm equilibrium condition, the planner solution can be implemented as an equilibrium iff
$$\lambda_1\pi_1 \geq \lambda_2\pi_2 + \frac{r}{r+(N-1)(r+\lambda_1)}\left(\frac{\lambda_1}{r+\lambda_3}\right)[\lambda_2\pi_2 - \lambda_3\pi_3].$$

Proposition 9 says that the planner may work on invention 1 even when $\lambda_1\pi_1 < \lambda_2\pi_2$ as long as doing so makes available a third invention with even higher expected flow payoff and the future is not discounted too heavily. We prove this by examining all six permutations of flow immediate payoff across the inventions.

Proof. In the cases:

$$\lambda_1\pi_1 \geq \lambda_2\pi_2 \geq \lambda_3\pi_3, \quad \lambda_1\pi_1 \geq \lambda_3\pi_3 \geq \lambda_2\pi_2, \quad \lambda_2\pi_2 \geq \lambda_1\pi_1 \geq \lambda_3\pi_3,$$

the solution (for both planner and firms) is to discover in decreasing order of $\lambda_i\pi_i$, since the graph does not impose any binding constraints. This can be shown directly with Proposition 2.

Consider the following cases:

- Case (a): $\lambda_3\pi_3 \geq \lambda_1\pi_1 \geq \lambda_2\pi_2$.
- Case (b): $\lambda_3\pi_3 \geq \lambda_2\pi_2 \geq \lambda_1\pi_1$.
- Case (c): $\lambda_2\pi_2 \geq \lambda_3\pi_3 \geq \lambda_1\pi_1$.

In these cases, the planner optimum may involve working on 1 first in order to "open up" valuable invention 3. In case c, by Proposition 4, we know the planner works on 2 before 3 conditional on inventing 1. The planner would invent 1 before 2 if and only if

$$p_1\pi_1 + p_1p_2\pi_2 + p_1p_2p_3\pi_3 \geq p_2\pi_2 + p_1p_2\pi_1 + p_1p_2p_3\pi_3,$$

Algebraic manipulation shows this condition is equivalent to $\lambda_1\pi_1 \geq \lambda_2\pi_2$. Therefore, the planner will always work on the project with the highest available flow profit and therefore we can implement the planner solution as a equilibrium under the baseline policy.

Consider now cases (a) and (b). By Proposition 4, we know the planner will work on $3 \to 2$ after discovering 1. Therefore, the planner will first invent 1 if and only if

$$p_1\pi_1 + p_1p_3\pi_3 + p_1p_2p_3\pi_2 \geq p_2\pi_2 + p_1p_2\pi_1 + p_1p_2p_3\pi_3,$$

Moving terms around and multiplying the expression by $\frac{r}{(1-p_1)(1-p_2)} = \frac{(r+\lambda_1)(r+\lambda_2)}{r}$ we get

$$\lambda_1\pi_1 \geq \lambda_2\pi_2 + \left(\frac{\lambda_1}{r+\lambda_3}\right)[\lambda_2\pi_2 - \lambda_3\pi_3]$$

Now, using the result about equilibrium implementation of the planner solution we get the statement in the proposition. □

6.5 Spillovers

In the main results, under the baseline policy, inventing firms collect the entire immediate social payoff of their invention, and non-inventing firms collect zero. Consider a policy where only a fraction α of the immediate social payoff is collected by inventors, with the remaining surplus accruing to all other firms, shared equally.

Definition 10. *Let a* spillover policy $\mathcal{P}_\alpha$ *provide inventors transfers* $w(s,s') = \pi(s,s')(1-(N-1)\alpha)$ *and noninventors* $z(s,s') = \alpha\pi(s,s')$*, Assume that* $\alpha \leq \frac{1}{N}$*, meaning inventors receiveweakly more than non-inventors.*

From proposition 2, it is easy to see that the distortions can be written as

$$D_\alpha(s,s') = D_{BC}(s,s') - (N-1)\alpha(\lambda_\ell\pi_\ell - \lambda_{s'}\pi_{s'}) + \mathcal{V}(\alpha) = (1-\alpha)D_{BC}(s,s') + \mathcal{V}(\alpha)$$

where $\mathcal{V}(\alpha)$ is the distortion from the difference between the social continuation value under the baseline policy and spillover policy $\mathcal{P}_\alpha$. Thus, letting non-inventors get a share of the immediate payoff weakens the directional distortion caused by the baseline policy.

6.6 Short Run vs Long Run Firm Equilibrium

In the main results, we look only at homogenous, infinitely-lived firms with perfect information about parameter values. Much of the intuition in those results can be generalized. In this subsection, let there be one long run innovator who plays until everything is discovered, and a sequence of short run innovators who play only one period each. Short run players may be R&D firms who only have the technological ability to work on exactly the present set of invention opportunities; they hence put no weight on the social value created when their inventions open up future opportunities for other firms.

Consider an invention graph with two technologically independent inventions. Let the total number of scientists $M = 1$, with the long run and the short run firm both having $\frac{1}{2}$ scientist. Since the number of scientists is constant, just as in the case of technologically independent inventions the planner works first on 1 rather than 2 if and only if $\lambda_1\pi_1 \geq \lambda_2\pi_2$.

The long run firm has the same best response as in the technologically independent inventions case since the identity of the rivals is irrelevant. The short run innovator at any stage has the best response:

$$s' \in \arg\max_{\tilde{s}\in S(s)} \frac{\lambda_{\tilde{s}}\pi_{\tilde{s}}}{N(r + \sum_{z\in S(s)} a_{-iz}\lambda_z) + \lambda_{\tilde{s}}}$$

The continuation values for the long run player are

$$V(1) = \frac{\lambda_2 \pi_2}{2r + 2\lambda_2}$$

and

$$V(2) = \frac{\lambda_1 \pi_1}{2r + 2\lambda_1}$$

Suppose the long run firm initially works on invention 1. The short run firm, when both inventions are available, works on invention 1 if and only if:

$$\lambda_1 \pi_1 \geq \lambda_2 \pi_2 + \frac{\lambda_2 \pi_2 (\lambda_1 - \lambda_2)}{2r + \lambda_1 + \lambda_2} \Leftrightarrow \frac{\lambda_1 \pi_1}{\lambda_2 \pi_2} \geq 1 + \Delta_1.$$

Suppose the long run innovator initially works on 2. The short run innovator when both inventions are available works on 1 if and only if:

$$\lambda_1 \pi_1 \geq \lambda_2 \pi_2 + \frac{\lambda_2 \pi_2 (\lambda_1 - \lambda_2)}{2r + 2\lambda_2} \Leftrightarrow \frac{\lambda_1 \pi_1}{\lambda_2 \pi_2} \geq 1 + \Delta_2,$$

where $\Delta_2 > \Delta_1$ as long as $\lambda_1 \neq \lambda_2$.

Therefore, when $\lambda_1 = \lambda_2$, there is no inefficiency. When $\lambda_1 < \lambda_2$,

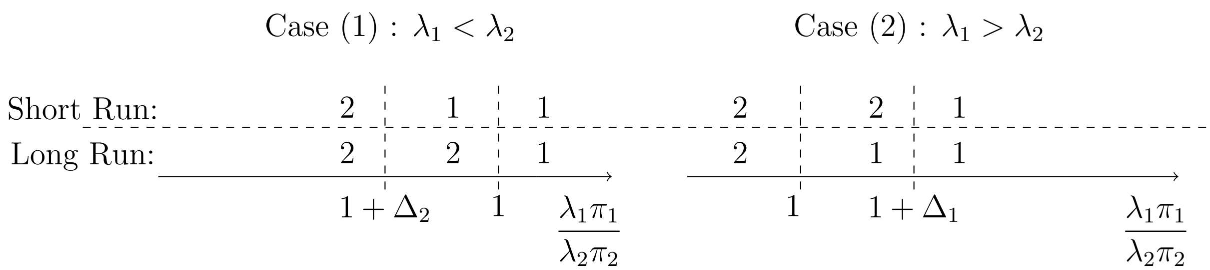
- If $\frac{\lambda_1 \pi_1}{\lambda_2 \pi_2} \geq 1$: Both long and short run firms working on 1 is an equilibrium (and it is efficient).
- If $\frac{\lambda_1 \pi_1}{\lambda_2 \pi_2} \leq 1 + \Delta_1$: Both working on 2 is an equilibrium (and it is efficient)
- When $1 + \Delta_1 \leq \frac{\lambda_1 \pi_1}{\lambda_2 \pi_2} \leq 1$, the short run and long run firm working on 1 is not an equilibrium. In this case, the equilibrium is asymmetric, hence inefficient.

Analogous conditions hold if $\lambda_1 > \lambda_2$.

The equilibrium is depicted in the following figure, where Δ_2 is negative and Δ_1 is positive.

It may seem counterintuitive that short run players deviate to the harder project. The short run player puts no value on being able to work on a second project after the first invention is completed. When the long run player works on the easy project first, a deviation by a short run player to the hard project delays the total expected time until

Figure 5: Equilibrium project choice with sequence of short run firms and a long run firm



both projects are completed. Since the short run player receives no continuation value, he completely ignores the harm of delaying the completion of both projects. Note how extreme this effect is: short run firms can work on a project in equilibrium even when it has a strictly lower flow immediate payoff than the social optimum.